Transmission Line Equations and Traveling Waves

Note Title 2/17/2014

*Definitions:

* Lumped parameters: size is very small compered to the applied signal's wave Length. (concentrated)

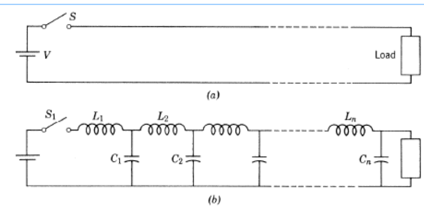
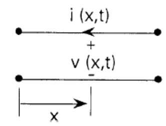


Fig. 9.1. (*a*) Two-wire transmission line. (*b*) "Lumpy" representation of a two-wire line.

* Distributed parameters: the quantities are distributed over tens or hundreds of miles. (Transmission Lines)



Circuit Representation

$$\frac{\partial i(x,t)}{\partial x} = C\frac{\partial v(x,t)}{\partial t}$$

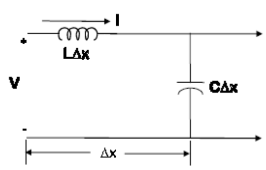
$$\frac{\partial v(x,t)}{\partial x} = L\frac{\partial i(x,t)}{\partial t}$$

Model

* After a time Δt a Length Δx of the line has been charged:

$$Q = CV\Delta x$$

$$I = \frac{dQ}{dt} \Rightarrow I = VC\frac{\Delta x}{\Delta t} = VC\frac{dx}{dt} = VC\nu$$



* Similarly, when the current has penetrated Δx, the flux ϕ is:

$$\phi = L\Delta x I = L\Delta x CV\nu$$

$$emf = \frac{d\phi}{dt} = LCV\nu\frac{\Delta x}{\Delta t} = LCV\nu^2 = V$$

$$\Rightarrow \nu = \frac{1}{\sqrt{LC}} \longrightarrow \text{velocity of the waves propagation}$$

in per unit Length

* Alternatively stated:

$$I = CV\nu = \frac{CV}{\sqrt{LC}} \quad \text{or} \quad \frac{V}{I} = \sqrt{\frac{L}{C}} = Z_0 \quad \text{characteristic impedance}$$

For overhead lines

- $225\,\Omega < Z_0 < 450\,\Omega$
- $\nu \approx c \approx 3 \times 10^8$ meters/second (speed of light)

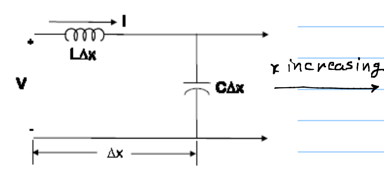
For cables

- $25\,\Omega < Z_0 < 60\,\Omega$
- $\nu \approx$ 1/3 to 1/2 c

Surge impedance by voltage range:

System kV	O/H Line Ω	Cable Ω
115	375	28
230	365	35
345	290	42
500	250	-

* The Wave Equation:



* The voltage across this section:

$$-\Delta V = L\,\Delta x \frac{\partial I}{\partial t}$$

$$\frac{\partial V}{\partial x} = -L \frac{\partial I}{\partial t} \longrightarrow (1)$$

* The current to charge ΔC is:

$$-\Delta I = C\,\Delta x \frac{\partial V}{\partial t} \longrightarrow (2)$$

Solving for the voltage

differentiate (1) w.r.t x:

$$\frac{\partial^2 V}{\partial x^2} = -L \frac{\partial^2 I}{\partial t \partial x}$$

differentiate (2) w.r.t t:

$$\frac{\partial I}{\partial t \partial x} = -C \frac{\partial V}{\partial t}$$

$$\frac{\partial^2 V}{\partial x^2} = LC \frac{\partial^2 V}{\partial t} \longrightarrow (3)$$

(3) & (4) are the wave equations

If we solve for the current:

$$\frac{\partial^2 I}{\partial x^2} = LC \frac{\partial^2 I}{\partial t^2} \longrightarrow (4)$$

* The general solution of (3) & (4):

$$I = f\left[x \pm \frac{1}{\sqrt{LC}}\right] \Rightarrow I(x,t) = f_1(x - vt) + f_2(x + vt)$$

$$V(x,t) = Lv\,[f_1(x - vt) - f_2(x + vt)]$$

$$= Z_0[f_1(x - vt) - f_2(x + vt)]$$

* If V and I waves are traveling in +x, they have same signs.

* If they travel in the negative direction, they have opposite signs.

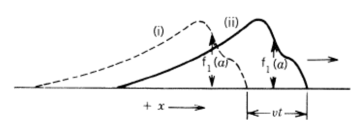
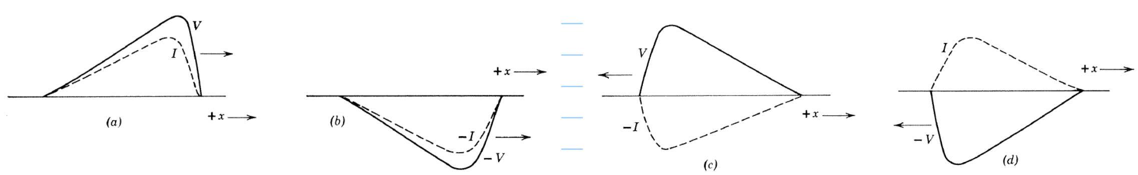


Fig. 9.4. The function $(x - vt)$ at (i) $t = 0$ and (ii) $t = \tau$.



*Another forms:

$$I(x,t) = f_1\left(\frac{x}{v} - t\right) + f_2\left(\frac{x}{v} + t\right), \quad V(x,t) = Z_0 f_1\left(\frac{x}{v} - t\right) - Z_0 f_2\left(\frac{x}{v} + t\right)$$